

A short mathematical description of Keynes's general theory of unemployment, prices and interest

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1 Setup

1.1 Income

Let y be the income - or total production of the economy, measured in some real production unit. It is assumed that the income is uniquely determined by the number of persons employed, n :

$$y = y(n),$$

where the function y is monotonically increasing. A certain part of the income is consumed,

$$c = c(y),$$

where the function c is monotonically increasing, but its growth is slow enough so that the surplus of income over consumption,

$$s(y) = y - c(y)$$

is increasing in y as well.

The expected profitability of investment is described by the marginal efficiency of capital:

$$e = e(v, q).$$

This is the expected rate of return on the least profitable investment when the income of the investment sector is v . It is assumed that investments are undertaken in order of decreasing efficiency, so that e is decreasing in v . The other argument is determining the expected profit is the level of optimism of investors, q which is defined so that e is increasing in q .

1.2 Wages and prices

Production cost, in money units, per unit of output - i.e., money wages and the cost of other production factors - increases with employment:

$$p = p(n).$$

This is a result of the increasing bargaining power of labor as well as the non-homogeneous nature of labor and other production factor. As income increases various production bottlenecks are encountered and as a result the cost of the production unit increases. At full employment, the function approaches a vertical asymptote.

Since income is an increasing function of employment wages can also be expressed as an increasing function of income:

$$p = p(y).$$

1.3 Monetary

The money in the economy m is used either for conducting transactions or for storing wealth. The quantities used for those purposes are m_{T} and m_{S} . Thus:

$$m = m_{\text{T}} + m_{\text{S}}.$$

The amount of money used for transactions depends linearly on income and on wages:

$$m_{\text{T}} = ap(y)y,$$

while the amount of money used for storing wealth depends on the interest rate and on production unit cost:

$$m_{\text{S}} = m_{\text{S}}(r, p).$$

It is decreasing in r and increasing in p .

2 Analysis

The optimism of the investors, q , and the total amount of money, m , are assumed exogenous.

Equilibrium is the point at which

$$e(v, q) = r.$$

Thus, the equation determining the equilibrium state is:

$$m = ap(y)y + m_{\text{S}}\left(e(s(y), q), p(y)\right),$$

with a single unknown, y . The right hand side is monotonically increasing in y since both summands are increasing in y . Since $p(y)$ increases asymptotically as unemployment decreases, a solution exists for any m such that

$$m \geq m_S(e(0, q), p(0)).$$

Otherwise, no solution exists and liquidity preference is absolute. The solution, $y(m, q)$, is increasing in both arguments.